**AP CS: Number Bases**

We can write the integer 7,965,841 as 7,000,000 + 900,000 + 60,000 + 5,000 + 800 + 40 + 1. We can write this statement as a result of the fact that our number system is base 10, meaning we have 10 digits. Each digit represents a multiple of a power of 10, based on its position. To be clearer, we could write

7,965,841= 7 X 106 + 9 X 105 + 6 X 104 + 5 X 103 + 8 X 102 + 4 X 101 + 1 X 100

The reason we count the way we do with a 9 followed by a new number consisting of a 0 and a 1 (10) is that we have ten fingers. If we were cartoon characters with only 8 fingers we might count using base 8. Then, we might only use 8 digits 0, 1, 2, 3, 4, 5, 6, and 7. To go higher than 7, we would create and “eights” place, so that 25 in our new number system would represent two 8’s and 5 one’s, or 2(8) + 5(1) = 21 in the base ten system. Higher positions would correspond to higher powers of 8; for example, 6543 means 6X83 + 5X82 + 4X81 + 3X80. To eliminate the confusion of going back and forth between two bases, we use notation 4710 means base 10 number 47 and 478 means 47 base 8 number 47. This notation carries over to other bases as well.

**You Try:** Convert the following to base 10. Show the work below- No calculators!

1. 478
2. 1123
3. 100112
4. 43215
5. 100335
6. 1212014
7. 9911

The presence of base 16 raises a new question: what if we want to use a base greater than 10? We will need more digits than the usual 10, so all we do is use some other symbols. The most common case is base 16, or hexadecimal. Here se use the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F. Thus A16 = 1010 and F16 = 1510. If we had a high enough base we might have to start using smiley faces and triangles for digits, but there would be little use for such a system.

**You Try:**

Find the base 10 equivalent for the following.

1. BEE16
2. DEF16
3. A116

At the opposite extreme from all these digits is the lowly base 2, or binary. Here the only two digits are, 0 and 1 and counting looks like 1, 10, 11, 100, 101, 110, …

Find the base 10 equivalent to the following binary numbers.

1. 111
2. 1011
3. 100001
4. 10101
5. 1101
6. Try to add the following binary numbers without converting them to base 10. 11001 + 1011